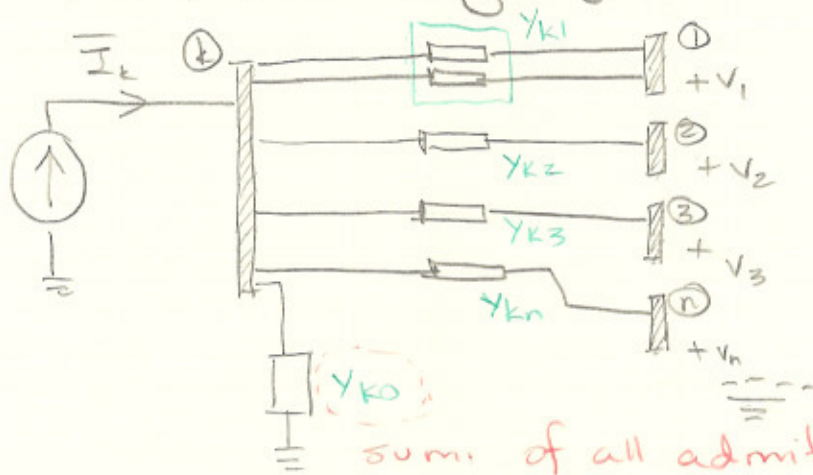


SYSTEM EQUATIONS

consider bus k of a system supplied by a current source \bar{I}_k and is connected through admittances to other buses, including ground as shown below



at bus k , kirchoffs current law, we have

$$\bar{I}_k = \bar{I}_{k0} + \bar{I}_{k1} + \bar{I}_{k2} + \dots + \bar{I}_{kn}$$

\bar{I}_k : source current injected into bus k

\bar{I}_{kj} : current flowing from node k to node j

In any branch or group of parallel branches connected from k to j , the voltage drop is $V_k - V_j$

And we can write

$$\bar{I}_{kj} = \bar{Y}_{kj}(V_k - V_j)$$

where \bar{Y}_{kj} is the sum. of admittance from bus k to bus j .

\bar{V}_k, \bar{V}_j are voltages measured wrt ground
combining the above we get

$$\bar{I}_k = \sum_{\substack{j=0 \\ j \neq k}}^n \bar{Y}_{kj} (\bar{V}_k - \bar{V}_j) \quad (3)$$

assume k is a number between 0 and j

for $k=2, n=3$, then

$$\bar{I}_2 = \bar{Y}_{20} \bar{V}_2 + \bar{Y}_{21} (\bar{V}_2 - \bar{V}_1) + \bar{Y}_{23} (\bar{V}_2 - \bar{V}_3)$$

$$\bar{I}_2 = (\bar{Y}_{20} + \bar{Y}_{21} + \bar{Y}_{23}) \bar{V}_2 - \bar{Y}_{21} \bar{V}_1 - \bar{Y}_{23} \bar{V}_3$$

equivalent to
 Y_{22} in Y_{bus} .

the above equation can be written in compact form

$$\bar{I}_2 = \bar{V}_2 \sum_{\substack{j=0 \\ j \neq 2}}^3 \bar{Y}_{kj} - \sum_{\substack{j=1 \\ j \neq 2}}^3 \bar{Y}_{2j} \bar{V}_j$$

Equation 3 given earlier as, can be written as-

$$\bar{I}_k = \bar{V}_k \sum_{\substack{j=0 \\ j \neq k}}^n \bar{Y}_{kj} - \sum_{\substack{j=0 \\ j \neq k}}^n \bar{Y}_{kj} \bar{V}_j$$

this equation can be written for every bus node except the reference (ground)

$$\left. \begin{aligned} \bar{I}_1 &= y_{11} \bar{V}_1 + \dots + y_{1n} \bar{V}_n \\ \bar{I}_2 &= y_{21} \bar{V}_1 + \dots + y_{2n} \bar{V}_n \\ &\vdots \\ \bar{I}_n &= y_{n1} \bar{V}_1 + \dots + y_{nn} \bar{V}_n \end{aligned} \right\} \text{can be expressed as matrix.}$$

where

\bar{Y}_{kj} : - (sum of admittances connected from k to j)

\bar{Y}_{kk} : (sum of all admittances connected to node k)

in power system problems, the source current are not known quantities. What is known at most buses is the complex power, then

$$\bar{S}_k = \bar{V}_k \bar{I}_k^*$$

$$\bar{I}_k = \frac{\bar{S}_k^*}{\bar{V}_k^*} = \frac{P_k - jQ_k}{\bar{V}_k^*} \quad (7)$$

where, \bar{S}_k is the complex power injected into bus k and is positive for a source at bus k and negative for a load at bus k

eqn 7, takes the form,

$$\frac{\bar{S}_k}{\bar{V}_k} = \sum_{j=1}^{\infty} \bar{Y}_{kj}^* \bar{V}_j^*$$

note:

$$\bar{S}_k = P_k + jQ_k = \bar{V}_k \bar{I}_k^*$$

$$\bar{S}_k = \bar{V}_k \sum_{j=1}^{\infty} \bar{Y}_{kj}^* \bar{V}_j^*$$

In polar form

$$\bar{V}_k = |\bar{V}_k| \angle \delta_k = |\bar{V}_k| e^{j\delta_k}$$

$$\text{or } \bar{V}_k = |\bar{V}_k| [\cos \delta_k + j \sin \delta_k]$$